

# Computation of Shortest path using Network Simplex Algorithm in Wireless Multicast Network

<sup>1</sup>Deepa V B, <sup>2</sup>UshaDevi M B

<sup>1</sup>Department of Information Science and Department of Telecommunication

<sup>2</sup>Jawaharalal Nehru National College of Engineering, Shimogga

---

**Abstract:** The construction of multicast tree within given constraints, such as delay and capacity, is becoming a major problem in many wireless multicast networks. The paper's contributions are to investigate the computational complexity of solving the Shortest path problem among different network shortest path algorithms and network simplex method and revised network simplex method. The network simplex algorithm is one of the most popular algorithms in practice for solving the minimum cost flow problem. This algorithm is an adaptation for the minimum cost flow problem of the well-known simplex method of linear programming. This result permits us to restrict our search for an optimal solution among spanning tree. The algorithm is flexible in the sense that we can select the entering arc in a variety of ways and obtain algorithms with different worst-case and empirical attributes tree solutions.

**Keywords:** Multicast, Shortest Path, Network Simplex Algorithms.

---

## 1. INTRODUCTION

Multicasting in wireless networks has been intensively studied by using several heuristics algorithms. Multicast as a means of communication is an essential part of many next generation networks and there are limited network layer that support multicast in the Internet today. According to [1], multicasting is the transmission of messages in a group of nodes which is recognized by one and unique address. Multicast services allow one source to send information to a large number of receivers and it finds application in many areas such as audio conferencing, video conferencing, video-on-demand etc. [2, 3]. Handling group communication is a key requirement for numerous applications that have one source which sends the same information concurrently to multiple destinations. Multicast routing refers to the construction of a tree rooted at the source and spanning all destinations. Generally, there are two types of such a tree, the Steiner tree and the shortest path tree. Steiner tree or group shared tree tends to minimize the total cost of the resulting tree, this is an NP-Complete problem. Number of heuristics to this problem can be found [4]. Shortest path tree or source-based tree tends to minimize the cost of each path from source to any destination, this can be achieved in polynomial time by using one of the two famous algorithms of Bellman [5] and Dijkstra [6], and pruning the undesired links. Recently, with the rapid evolution of multimedia and real time applications like audio/video conferencing, interactive distributed games and real-time remote control system, certain QoS need to be guaranteed in the resulted tree. One such QoS is the end-to-end delay between source and each destination, where the information must be sent within a certain delay constraint D. By adding this constraint to the original problem of multicast routing, the problem is reformulated and the multicast tree should be either delay-constrained Steiner tree, or delay constrained shortest path tree. Delay constrained Steiner tree is an NP-Complete problem [7], several heuristics are introduced for this problem [7][8][9] each trying to get near optimal tree cost, without regarding to the cost of each individual path for each destination. Delay constrained shortest path tree is also an NP Complete problem [10]. An optimal algorithm for this problem is presented at [9], but its execution time is exponential

and used only for comparison with other algorithms. Heuristics for this problem is presented in [10], which tries to get a near optimal tree from the point of view of each destination without regarding the total cost of the tree.

## 2. RELATED WORK

There are a number of multicast routing proposals in the existing literature for single as well as multiple channels for a WMN [11–19]. Jahanshahi et al. [11] proposed a cross-layer design for joint channel assignment and multicast tree construction problem in WMNs. A comprehensive cross-optimization framework based on the binary integer programming (BIP) formulation of the problem is presented which also addresses the hidden channel problem in WMNs. Karimi et al. [12] proposed an iterative primal–dual optimization framework for multicasting based on Lagrange relaxation for throughput maximization in WMNs. Authors in [13] have proposed a multicast routing algorithm for WMNs. It builds a Steiner tree and multicast data packet using gateway nodes. But the scheme proposed in [13] only works when the Internet Service Provider (ISPs) sets the prefix for all the gateways and mobile network subscribers. A mesh client node without a prefix will not have the authority to register with the gateway as proposed in [13]. These algorithms try to minimize the interference among multicast nodes in a routing tree in order to maximize the throughput of the multicast group. Lim et al. [14] proposed an efficient multicasting for multichannel WMNs. Nguyen [15] proposed two multicast algorithms as the Level Channel Assignment (LCA) algorithm and the Multichannel Multicast (MCM) to improve the throughput for multichannel and multi-interface WMNs. These algorithms build efficient multicast trees by minimizing the number of relay nodes and total hop count distances of the trees. Also, they use dedicated channel assignment strategies to reduce the interference to improve the network capacity. Zeng et al. [16] proposed three separate algorithms for multicasting based on three intelligent computational methods such as genetic algorithm, simulated annealing, and tabu search. The proposed algorithms aim to search minimum-interference multicast trees which also satisfy the end-to-end delay constraint and optimize the usage of the network resources. Li et al. [17] proposed two new load aware routing metrics FLMM and FLMM R to solve multicast throughput optimization problem. Authors have considered concurrent multicast flows and show that the throughput can be improved by seeking the multicast route with lower channel congestion degree. Pourfakhar [18] proposed a new neural network model to predict route or node disconnection to control congestion and losses specifically in the gateways. This prediction leads to recover before fault occurrence. In the same work, authors also proposed a new QoS multicast routing framework for WMNs to solve the problem of load balancing and to enhance the QoS in multicast communication among Internet hosts and mesh hosts dynamically. Prashanth et al. [19] proposed an algorithm, MARS which is distributed in nature, and relies on local network measurements to select a transmission bit-rate for a given multicast group. The proposed algorithm facilitates the joint use of bit-rate selection and link-layer mechanisms such as acknowledgments and retransmissions to improve reliability of high throughput multicast streams. Han and Guo [20] have studied the problem of collision-free multicast routing in multi-interface multi-channel wireless mesh networks, and present two heuristic algorithms with the aim of reducing both the interface redundancy and the multicast latency. Authors in [21] have proposed a novel approach using traffic engineering to enhance the performance of QoS multicast routing algorithms. A prioritized admission control scheme is used for optimal consumption of bandwidth in different connections in a multicast session. A Probabilistically Reliable Multicast Routing (PRMR) algorithm based on link cost is proposed in [22]. Moreover, a QoS aware multicast routing algorithm is proposed in [23]. Torkestani and Meybodi [24] proposed a weighted routing algorithm in which the mobility parameters are supposed to be random variables with unknown distribution. In the proposed scheme, the multicast routing problem is first transformed into an equivalent stochastic Steiner tree problem. A learning automata-based algorithm is proposed for solving the proxy Steiner tree problem in the proposed scheme.

## 3. NETWORK MODEL

The communication network is modelled as a directed, simple, connected weighted graph  $G=(V,E)$ , where  $V$  is the set of nodes and  $E$  is the set of directed links. Each link  $e$  in  $E$  connects two nodes  $u, v$  in  $V$  and is represented as  $e(u,v)$ . Two non-negative real value functions are associated with each link, the cost function  $Cost(u,v)$  represents the utilization of the link and the delay function  $Delay(u,v)$  represents the delay that the packet experiences through passing that link including switching, queuing, transmission and propagation delays. Links are asymmetrical, i.e.  $Cost(u,v)$  and  $Delay(u,v)$  do not necessarily equal  $Cost(v,u)$  and  $Delay(v,u)$ . A sequence of links that connects two nodes  $u, v$  are represented by a  $Path(u,v)$  with  $Cost(Path(u,v))$  which is equal to the sum of the costs of all its links, and  $delay(Path(u,v))$  which is equal to the sum of the delays of all its links. Multicast group  $M \subseteq V$  is a set of nodes that receives packets from source  $S \in V$ . The

least cost tree (LC) is a tree originating at the source  $S$  and spanning all members of  $M$  with minimum cost for each of them individually. The least delay tree (LD) is a tree originating at the source  $S$  and spanning all members of  $M$  with minimum delay for each of them individually. Using the previous notations and definitions, the problem of shortest path multicast routing under delay and delay variation constraints can be formulated as follows: Given a directed, simple, connected weighted graph  $G(V, E)$ , multicast group  $M \subseteq V$ , a multicast source node  $s \in V$ , a delay constraint  $\Delta$ , and a delay variation constraint  $\delta$  where  $\delta < \Delta$ . Find a subgraph  $G'(V', E')$  where  $(\{s\} \cup M) \subseteq V' \subseteq V$  and  $E' \subseteq E$  that has no cycles and no incoming edge in the source node  $s$  and all the leaves should be members of  $M$ .  $G'$  should satisfy the following three conditions:

1- Minimum  $Cost(Path(s, v)) \forall v \in M$

2-  $Delay(Path(s, v)) < \Delta \forall v \in M$

3-  $Delay(Path(s, u)) - Delay(Path(s, v)) < \delta \forall u, v \in M$

This problem is known to be NP-Complete. So, any polynomial solution tends to find a near optimal solution. If  $\delta > \Delta$ , the problem is reduced to the delay constrained shortest path problem.

#### 4. NETWORK SIMPLEX METHOD

At the beginning of each iteration we have a strongly feasible tree  $T$  and an associated basic flow vector  $x$  such that

$$x_{ij} = 0, \quad \forall (i, j) \notin T \quad 1$$

and a price vector  $p$  such that

$$r_{ij} = a_{ij} + p_j - p_i = 0, \quad \forall (i, j) \in T$$

The iteration has three possible outcomes:

- We will verify that  $x$  and  $p$  are primal and dual optimal, respectively.
- We will determine that the problem is unbounded.
- We will obtain by the method that, a strongly feasible tree  $T = T + e - e'$ , differing from  $T$  by the in-arc  $e'$  and the out-arc  $e$ .

*Typical Simplex Iteration*

Select an in-arc  $e = (i, j) \in T$  such that

$$r_{ij}^- = a_{ij}^- + p_j^- - p_i^- < 0$$

(If no such arc can be found, terminate;  $x$  is primal-optimal and  $p$  is dual optimal.) Consider the cycle  $C$  formed by  $T$  and  $e$ . If  $\bar{C}$  is empty, terminate [7] (the problem is unbounded); else, obtain the out-arc  $e' \in \bar{C}$

A Shortest Path Example

The shortest path problem is to find the directed paths of shortest length from a given *root* node to all other nodes [27]. We will use this example to illustrate the simplex method and some of its special properties when applied to shortest path problems. The corresponding minimum cost flow problem is

minimize,

$$\sum_{(i,j) \in \mathcal{A}} a_{ij} x_{ij}$$

subject to

$$\sum_{\{j|(1,j) \in \mathcal{A}\}} x_{1j} - \sum_{\{j|(j,1) \in \mathcal{A}\}} x_{j1} = 3,$$

$$\sum_{\{j|(i,j) \in \mathcal{A}\}} x_{ij} - \sum_{\{j|(j,i) \in \mathcal{A}\}} x_{ji} = -1, \quad i = 2, 3, 4$$

$$0 \leq x_{ij}, \quad \forall (i, j) \in \mathcal{A}.$$

The optimum solution to this problem sends unit flow from the root to every other node along a shortest path.

#### 4.1 Revised Simplex Method

Benefit of revised simplex method is clearly comprehended in case of large LP problems[26]. In simplex method the entire simplex tableau is updated while a small part of it is used. The revised simplex method uses exactly the same steps as those in simplex method. The only difference occurs in the details of computing the entering variables and departing variable. In standard simplex, the most computation time would be due to updating every column in each iteration. Even though this assures the availability of all data needed for the next pivot (pivot position not yet determined). We actually end up using only one column to make the decision on which variable will exit the basis.

An array that we need:

– For the simplex method:  $(m + 1) \times (n + 1)$

– For the revised simplex:  $(m + 1) \times (m + 2)$

If  $n$  is significantly larger than  $m$ , this would result in a substantial saving in computer core storage. The number of multiplications (division is considered a multiplication) and additions (subtraction is considered an addition) per iteration of both procedures are given in below:

**Table 1 Operation Required for Iterations**

Method	Pivoting	$Z_j - C_j$	Total
Simplex	Multi: $(m+1)(n-m+1)$ Add: $m(n-m+1)$		$m(n-m)+n+1$ $m(n-m+1)$
Reverse Simplex	Multi: $(m + 1)^2$ Add: $m(m + 1)^2$	$m(n-m)$ $m(n-m)$	$m(n-m)+(m + 1)^2$ $m(n+1)$

From the Table 1 we see that the number of operations required during an iteration of the simplex method is slightly less than those required for the revised simplex method.

#### 4.2 Computation Comparison

##### 1. Dijkstra's Algorithm

- Computational cost of Dijkstra's algorithm using an array or list to store the labelled vertices is  $O(V^2 + E) = O(V^2)$

##### 2. Bellman Ford algorithm

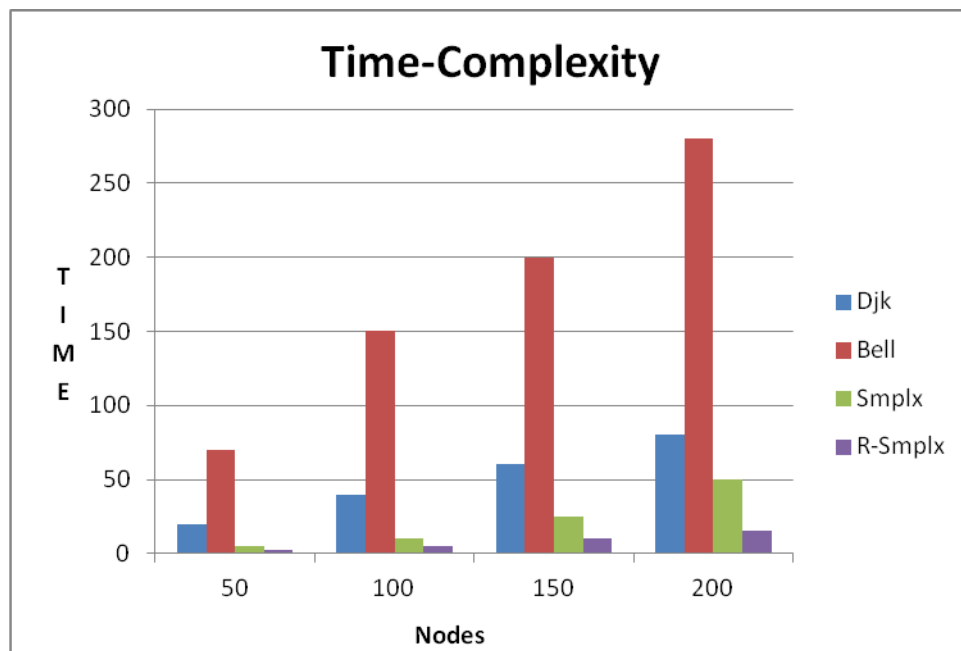
- In the worst case computational cost of Bellman Ford algorithm uses  $O(V^3)$  time in order to find single-source shortest paths. This is not very efficient. By a slight modification it can find all-pairs shortest paths in the same time.

##### 3. The Network Simplex Algorithm

An algorithm is said to be a polynomial-time algorithm if its running time is bounded by a polynomial function of  $n$ ,  $m$ ,  $\log C$ , and  $\log U$ . It is strongly polynomial if polynomial functions of  $n$  and  $m$  only, bound the running time, and pseudo-polynomial if polynomials bounded in terms of  $n$ ,  $m$ ,  $C$ , and  $U$ . If we require the removal of  $C$  and  $U$  from the bound then we invoke the *Similarity Assumption*. This is the assumption that both  $C$  and  $U$  are polynomials bounded in  $n$ , i.e. that  $C = O(n^k)$  and  $U = O(n^k)$ . The terms 'running time' and 'order of complexity' are used interchangeably.

*Polynomial-Time Bounds:*  $S(n, m, C) = \min\{m \log \log C, m + n\sqrt{\log C}\}$

*Strongly Polynomial-Time Bounds:*  $S(n, m) = m + n \log n$



**Figure 2 Time and Computation Complexity of Different**

#### Shortest path Algorithm

Simulations were carried out for 50,100,150,200 randomly generated nodes for multicasting to groups of receivers using ns2, the results is presented in Figure 2, and the cost of computation time of multicast in different shortest path algorithms were obtained and presented in graph as shown in Figure 2.

### 5. CONCLUSION

This paper introduces an idea of computing shortest path in wireless multicast network using network simplex algorithm comparing with revised simplex method. The Minimum Cost Flow Problem has a rich history, and it forms a framework upon which a wide range of theoretical and practical problems may be formulated and solved. There exist, at present, three dominant solution paradigms, namely the Network Simplex Method, Relaxation algorithms and Scaling algorithms. The superiority of any one method is a highly contentious issue. Simple pricing strategies, in general, out-perform their more complicated counterparts, mainly due to the elimination of expensive computational overhead comparing Dijiskra and Bellman Ford shortest path algorithm.

### REFERENCES

- [1] K. Chopra and A. Mishra, "Packet Rejection Based on Packet Rank for Congestion Control," *management*, vol. 1, 2012.
- [2] E. HariPriya and K. R. Valluvari, "A Network survey and a Comparative Study of Multicast Routing Techniques in Disconnected Adhoc," *International journal of scientific research engineering & Technology* vol. 2, pp. 237-241, 2013.
- [3] F. Wu, Y. Sun, Y. Yang, K. Srinivasan, and N. B. Shroff, "Constant Delay and Constant Feedback Moving Window Network Coding for Wireless Multicast: Design and Asymptotic Analysis," *arXiv preprint arXiv:1404.3438*, 2014
- [4] Hwang, F. and D. Richards 1992. "Steiner Tree Problems", *Networks*, vol. 22, no. 1, pp 55-89, January 1992
- [5] Bellman, R. 1952, *Dynamic Programming*. Princeton University Press, 1957.
- [6] Dossey, J.; A. Otto; L. Spence; and C. Eynden 1993 *Discrete Mathematics*, Second Edition,
- [7] Cunningham, W.H. (1976) 'A Network Simplex Method' *Mathematical Programming* Volume 11 p105-116.

- [8] Wi, S. and Y. Choi. 1995 "A Delay-Constrained Distributed Multicast Routing Algorithms", in *Proceeding of the twelfth International Conference on Computer Communication (ICCC '95)*, pp. 883-838.
- [9] Widyono, R. 1994 "The Design and Evaluation of Routing Algorithms for Real-Time Channels", Technical Report ICSI TR-94-024, University of California at Berkeley, International Computer Science Institute, [Winter 1987] Winter, P. 1987 "Steiner Problem in Networks: A Survey", *Networks*, vol. 17, no. 2, pp. 129-167.
- [10] Zhu, Q.; M. Parsa; and J. Garcia-Luna-Aceves. 1995 "A Source-Based Algorithm for Delay-Constrained minimum-Cost Multicasting", in *Proceeding of IEEE NFOCOM'95*, pp. 337- 385.
- [11] Y. Xiao, K. Thulasiraman, and G. Xue. Equivalence, unification and generality of two approaches to the constrained shortest path problem with extension. In *Allerton Conference on Control, Communication and Computing, University of Illinois*, pages 905–914, 2003.
- [12] O.B. Karimi, J. Liu, Z. Li, Multicast in Multi-channel Wireless Mesh Networking, in: *Lecture Notes in Computer Science, LNCS*, vol. 6091, 2010, pp. 148–159.
- [13] Z. Ke, L. Li, Q. Sun, N. Chen, A QoS multicast routing algorithm for wireless mesh networks, in: *Proc. of the ACIS International Conference on Software Engineering, Artificial Intelligence, Networking, and Parallel/Distributed Computing, SNPDC, 2007*, pp. 835–840.
- [14] S.-H. Lim, C. Kim, Y.-B. Ko, N.H. Vaidya, Efficient multicasting for multi-channel multi-interface wireless mesh networks, in: *Proc. of the IEEE Military Communications Conference, MILCOM, 2009*.
- [15] U.T. Nguyen, On multicast routing in wireless mesh networks, *Computer Communications* 31 (7) (2008) 1385–1399.
- [16] G. Zeng, B. Wang, Y. Ding, L. Xiao, M.W. Mutka, Efficient multicast algorithms for multichannel wireless mesh networks, *IEEE Transactions on Parallel and Distributed Systems* 21 (1) (2010) 86–99.
- [17] F. Li, Y. Fang, F. Hu, X. Liu, Load-aware multicast routing metrics in multi-radio multi-channel wireless mesh networks, *Computer Networks* 55 (9) (2011) 2150–2167.
- [18] E. Pourfakhar, A.M. Rahmani, A hybrid QoS multicast framework-based protocol for wireless mesh networks, *Computer Communications* 33 (17) (2010) 2079–2092.
- [19] Prashanth A.K. Acharya, Elizabeth M. Belding, MARS: link-layer rate selection for multicast transmissions in wireless mesh networks, *Ad Hoc Networks* 9 (1) (2011) 48–60.
- [20] K. Han, Q. Guo, Reducing multicast redundancy and latency in wireless mesh networks, in: *Proc. of the International Workshop on Education Technology and Computer Science, ETCS, 2009*, pp. 1075–1079.
- [21] B. Rong, Y. Qian, K. Lu, R.Q. Hu, Enhanced QoS multicast routing in wireless mesh networks, *IEEE Transactions on Wireless Communications* 7 (6) (2008) 2119–2130.
- [22] X. Zhao, C.T. Chou, J. Guo, S. Jha, A scheme for probabilistically reliable multicast routing in wireless mesh networks, in: *Proc. of the IEEE Conference on Local Computer Networks, LCN, 2007*, pp. 213–214.
- [23] L. Zhao, A.Y. Al-Dubai, G. Min, A QoS aware multicast algorithm for wireless mesh networks, in: *Proc. of the IEEE International Symposium on Parallel & Distributed Processing, IPDPS, May 2009*.
- [24] J.A. Torkestani, M.R. Meybodi, A link stability-based multicast routing protocol for wireless mobile ad hoc networks, *Journal of Network and Computer Applications* 34 (4) (2011) 1429–1440.
- [25] Rolando Menchaca-Mendez, J.J. Garcia-Luna-Aceves, Hydra: efficient multicast routing in MANETs using sender-initiated multicast meshes, *Pervasive and Mobile Computing* 6 (1) (2010) 144–157.
- [26] Cunningham, W.H. (1976) 'A Network Simplex Method' *Mathematical Programming* Volume 11 p105-116
- [27] Edmonds, J. and Karp, R.M. (1972) 'Theoretical Improvements in Algorithmic Efficiency for Network Flow Problems' *Association for Computing Machinery Journal* Volume 19 p248-264.